Developing Genetic Algorithms and Mixed Integer Linear Program Models for Finding Optimal Strategies for a Student's "Sports" Activity

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ABSTRACT

An important advantage of genetic algorithms (GAs) are their ease of use, their wide applicability, and their good performance for a wide range of different problems. GAs are able to find good solutions for many problems even if the model of the problem is accurate, realistic, and complicated. In contrast, classical optimization approaches like linear programming or mixed integer linear programming (MILP) can only be applied to restricted types of problems as non-linearities and and other realistic properties of a problem that occur in many real-world applications can not appropriately modeled.

This paper illustrates for an entertaining student "sports" game that GAs can easily be adapted to a problem of unknown properties and complexity and are able to solve the problem easily. Modeling the problem as a MILP and trying to solve the problem by using a standard MILP solver reveals that the problem is not solvable for such optimization methods whereas GAs can solve it in a few seconds. Furthermore, the effort for building a MILP model is much higher than developing a simple GA for the problem.

The game studied is known to students as the so-called "beer-run". There are different teams that have to walk a certain distance and to carry a case of beer. When reaching the goal all beer must have been drunken by the group and the winner of the game is the fastest team. The goal of optimization algorithms is to determine a strategy that minimizes the time necessary to reach the goal. This problem was chosen as it is not well studied and allows to demonstrate the advantages of using metaheuristics like GAs in comparison to standard optimization methods like MILP solvers for problems of unknown structure and complexity.

Categories and Subject Descriptors

I.2.8 [**Artificial Intelligence**]: Problem Solving and Search-Heuristics

Keywords

genetic algorithms, ease of use, MILP, discrete event simulation, games

1. INTRODUCTION

During the last couple of decades, computing power has – according to Moores Law [6] – increased enormously. Due to the large available computing power, optimization methods like genetic algorithms GAs [5, 7], simulated annealing [9], tabu search [4], and other metaheuristics [3] have been applied to a wide range of problems. The large benefit of such metaheuristics is their their ease of use and their good performance for a large number of different problems. For the application of metaheuristics it is enough to define a fitness function that assigns fitness values to different solutions for a problem and to develop a representation where search operators can be applied to. Therefore, many problems can be solved by metaheuristics that are not tractable by classical optimization approaches.

As it is difficult to compare the effort for applying different optimization algorithms to well-known optimization problems, in this paper we apply a mixed integer linear problem (MILP) solver and a GA to a problem of unknown properties. We have chosen an unconventional and entertaining problem and want to find optimal strategies for the so-called "beer-run". A beer run is a (strange) kind of leisure activity which is popular among college and university students. In a beer run there are different teams that have to walk a certain distance and who have to carry a case of beer. When reaching the goal all beer must have been drunken by the team and the winner of the game is the fastest team. Finding the optimal strategy, the strategy that minimizes running time, is a non-trivial optimization problem as various and contradicting effects during the walk have to taken into account. The "beer-run" is a representative example for opimization problems where people (for example firefighters) have to move in hazardous and poisonous environments (for example polluted air or dangerous smoke) that affects their physical abilities. On the one hand inhaling polluted air is necessary to survive but on the other hand it reduces the speed (and physical abilities) of the humans. The "beer-run" can be viewed as a formalized model for such situations and finding optimal strategies on when to inhale polluted air can be important.

The purpose of this paper is to examine the effort for developing different optimization algorithms (MILP and GA) for this problem. The considered game has not been investigated before, so we do not know specific properties of the problem nor its complexity. In the paper, we develop a MILP model for the problem and compare the effort for developing a MILP model to the effort for applying a simple GA to the problem. Furthermore, we study the performance of the different optimization approaches. The results reveal that the effort for developing an appropriate MILP model for the problem is much higher than the effort for developing a GA. Furthermore, for MILPs only simplified models are possible and many properties of real-world problems can not be modeled at all. The comparison of the performance of MILP solver and GAs reveals that MILP solver can not solve even simple versions of the problem whereas GAs find optimal solutions in a few seconds.

The paper is organized as follows: the following section introduces the optimization problem. It describes the problem, creates a model of the problem including objective function and input variables, and discusses intuitive optimal solutions for the problem. Furthermore, a brief comparison to similar problems is given. Section 3 develops a MILP model for the problem and Section 4 a GA. Section 5 presents experimental results and compares the performance of the different optimization methods. The paper ends with concluding remarks.

2. OPTIMIZATION PROBLEM

This section discusses the optimization problem and intuitive optimal solutions. To gain a better understanding of the problem is important for developing efficient optimization methods.

2.1 Generic Problem Description

In the so called "beer-run", a team of two people have to cover a certain distance carrying a case of beer with them. The goal is to cover the distance as fast as possible under the constraint that when reaching the goal all beer has to be drunken by the two people. Finding optimal strategies for the game is difficult as several counteracting effects occur. For example, due to having a beer the weight that has to be carried by the two persons decreases which leads to a speed increase. In contrast, the blood alcohol level (BAL) increases which leads to a lower speed of the person. Furthermore, there are tight restrictions for the objective function, since it is necessary to cover a given distance and to drink all bottles of beer while covering the distance.

There are several problems from other domains that are related to the problem at hand. Examples are lot-sizing problems like the dynamic joint replenishment problem [1, 2] where customer's demand has to be fulfilled by a warehouse. The decision variables for the warehouse (the amount of goods that are ordered at a certain time point) are equivalent to the decisions on how much beer has to be consumed at which point of time. Other similar problems are control problems in rocket science. The problem is to determine an optimal strategy for using the carried fuel to speed up the rocket. Decisions have to be made on the time points when and how much fuel is used to propel the rocket.

2.2 Objective Function and Variables

The goal of the game is to travel a given distance d as fast as possible. There are two persons that have to carry a case of beer with n^b bottles. Each bottle contains beer of weight c^b. At each time point t the person j can decide to drink a bottle of beer, or not. For $x_{t,j} = 1$, person j drinks a beer at time t. The maximum number of bottles that can be consumed by the two persons in the group are n^b. Drinking a bottle of beer has two effects. The weight c_t of the case decreases allowing the team to walk with a greater speed. $v_{t,j}$ indicates the speed of person j at time t. It is assumed that there is a maximum speed v_j^{max} for each person which is reached if no additional weight have to be carried. When carrying an additional weight, v_j^{max} is reduced by Δv_j^w for each additional kilogram of weight that has to be carried. As having a bottle of beer reduces the weight c_t of the case at time t, the speed $v_{t,j}$ of person j at time t increases. Furthermore, we assume that the two people carrying the case stay together and the speed of the team is equal to the speed of the slowest person. The second effect of drinking a bottle of beer is an increase of the blood alcohol level (BAL) which leads to a decrease of the speed of the person j that has drunken a bottle of beer. Δp_j denotes the increase of BAL (in parts per thousands) of the person j due to drinking one bottle of beer. The increase of the BAL leads to a decrease of the speed of the person. Δv_j^p denotes the decrease of the maximum speed v_j^{max} of person j if the blood alcohol level $p_{t,i}$ increases by one part per thousands (ppt). A summary of the different variables necessary for modeling the game is given in Table 1.

constants	
c^b	weight of beer in a bottle
n^b	number of bottles in a case
d	distance of the beer run
v_j^{max}	maximum speed of person j
Δv_j^w	decrease of speed v per kg for person j
Δv_j^p	decrease of speed per blood alcohol level (in ppt) for person j
Δp_j	increase of blood alcohol level (in ppt) for person j due to drinking one bottle of beer with weight c^b

time-dependent variables	
c_t	weight of case at time t
$v_{t,j}$	speed of person j at time t
$p_{t,j}$	blood alcohol level of person j
r_t	distance d is not traveled at time t (binary variable)
$w_{t,j}$	person j is slowest person of the team at time t (binary variable)

decision variable	
$x_{t,j}$	person j drinks a beer at time t

Table 1: Variables

The weight of the case c_t at time t and the BAL $p_{t,j}$ of the two persons have a different effect on the walking speed $v_{t,j}$. Having a beer will lead to a gradually rising BAL and thus a reduction of speed $v_{t,j}$. The reduction of speed does not occur immediately after having a beer, but there is some delay. The individual person's BAL depends on his weight and sex (men exhibit a significantly higher rate of body fluid leading to a lower BAL).

For calculating the impact of having a beer on the BAL, we assume that the alcohol level increases linearly over one hour after the consumption [10] and then remains at this level during the whole walking period (until both persons reach the goal). The peak alcohol level is calculated according to the Widmark-Formula [10] as

$$p = \frac{a}{mr}, \tag{1}$$

where p is the BAL, a is the consumed alcohol in g and r is a factor depending on the sex of the person (we use 0.8 for males and 0.7 for females [10]). a is calculated as the product of the density of alcohol ($0.8l/mg$), the volume share of alcohol (e.g. for beer about 5%), and the amount of drunken beer (usually a bottle contains 0.33 l of beer). We do not consider a decrease in BAL as the reduction of the alcohol in the blood by the human body starts about two hours after the consumption of alcohol. As the maximum time of the game is only three hours, the game is almost over when the BAL is reduced again and we can neglect the reduction of the BAL.

Consequently, a team of two "beer runners" has several decisions to take during the run. They have to decide when to have a drink (overall they have to drink n^b bottles) and who of the two team members drinks the bottle.

2.3 Solution Strategies

There are different intuitive strategies on how to win the game. Since it takes a certain time for the BAL to rise, one supposable strategy could be to drink as late as possible. This "delay strategy" is arguable though, since the team members have to carry the full weight of the case over the whole distance d.

Drinking as early as possible denotes the other extreme. This "weight-shifting strategy" is debatable as well, since the persons speed is reduced by the increasing BAL leading to a lower speed of the team.

Therefore, we will determine an optimal strategy using two different optimization algorithms for the problem. Consequently, the following Section presents an MILP formulation of the problem and Section 4 describes a GA design.

3. A MIXED INTEGER LINEAR MODEL

The problem of finding an optimal strategy can be formulated as an mixed integer linear problem. To model the problem, we introduce an additional binary variable r_t which indicates if the two persons have reached the goal after time t ($r_t = 0$ means they reached the goal at time t). Therefore, the objective of the MILP is to minimize the sum of all r_t.

The problem can be formulated as the following mixed integer linear model of the game:

$$\sum r_t \quad \rightarrow \quad \min \tag{2}$$

$$r_t - r_{t+1} \quad \geq \quad 0 \quad \forall t \tag{3}$$

$$r_t - x_{t,j} \quad \geq \quad 0 \tag{4}$$

$$c_{t-1} - \sum_j c^b x_{t,j} \quad = \quad c_t \quad \forall t \tag{5}$$

$$\sum_t \sum_j x_{t,j} \quad = \quad n^b \tag{6}$$

$$\sum_{u=t-60}^{t-1} \frac{\Delta p_j}{60} x_{u,j} + p_{t-1,j} \quad = \quad p_{t,j} \quad \forall t, j \tag{7}$$

$$v_j^{max} r_t - \frac{1}{2} \Delta v_j^w c_t - \Delta v_j^p p_{t,j} \quad = \quad v_{t,j} \quad \forall t, j \tag{8}$$

$$\frac{1}{\sum_j v_j^{max}} \times (v_{t,0} - v_{t,1}) - w_{t,1} \quad \leq \quad 0 \quad \forall t \tag{9}$$

$$\frac{1}{\sum_j v_j^{max}} \times (v_{t,1} - v_{t,0}) - w_{t,0} \quad \leq \quad 0 \quad \forall t \tag{10}$$

$$v_j^{max} \times w_{t,j} - y_{t,j} \quad \geq \quad 0 \quad \forall t, j \tag{11}$$

$$\frac{1}{v_j^{max}} \times v_{t,j} - \frac{1}{v_j^{max}} \times y_{t,j} - w_{t,j} \quad \leq \quad 0 \quad \forall t, j \tag{12}$$

$$\sum_t \sum_j y_{t,j} \quad \geq \quad d \tag{13}$$

$$c_0 \quad = \quad 15 \tag{14}$$

$$\sum_j w_{t,j} \quad = \quad 1 \quad \forall t \tag{15}$$

$$p_{0,j} \quad = \quad 0 \quad \forall j \tag{16}$$

$$x_{t,j}, r_{t,j}, w_{t,j} \quad = \quad \{0, 1\} \tag{17}$$

$$y_{t,j}, p_{t,j}, c_t \quad \geq \quad 0 \tag{18}$$

(2) is the objective function of the problem. $r_t = 1$ indicates that the two persons are still walking at time t and $r_t = 0$ indicates that they have reached the goal at time t. (3) ensures that once the two team members have reached the goal at time t_{goal}, $r_t = 0$ for $t > t_{goal}$. (4) ensures that the two team members can only reach the goal after they have drunken all bottles. (5) calculates the weight c_t of the case at time t. Each time a beer is drunken, the weight of the case is reduced by the weight c^b of one beer. (14) denotes that the initial weight of the case is 15 kg. (6) ensures that all n^b bottles of the case must be drunken by one of the two persons during the walk. The decision variables $x_{t,j} = 1$ if person j drinks a beer at time t. (7) calculates the BAL $p_{t,j}$ of person j at time t. The BAL at time t depends on the BAL at time $t-1$ and the number of beers that have been drunken in the last 60 minutes. After consuming a beer, the BAL increases in each minute for $\Delta p_j/60$ ppt as the consumption of a beer increases the BAL over the next 60 minutes to a maximum of Δp_j. After 60 minutes the maximum is reached and the BAL remains constant (if no more beers are consumed). (8) calculates the speed $v_{t,j}$ of person j at time t. The maximum speed v_j^{max} of person j is reduced by the weight c_t of the case at time t and the BAL of person j. It is important to notice that the case is always carried by both persons. Therefore, each person has only to carry half of the weight. (9) and (10) are necessary to determine the slower of the two persons in the team. $w_{t,0} = 1$ if person 0 is slower than person 1 and $w_{t,0} = 0$ if person 0 is not slower than person 1. (11) introduces a help variable $y_{t,j}$ which is set to $y_{t,j} = v_{t,j}$ if person j is the slower team member. Otherwise, if person j is the faster team member $y_{t,j} = 0$. (11) ensures that for $w_{t,j} = 0$ (person j is faster than the other person), the help variable $y_{t,j} = 0$. This means, the speed y_t of the faster team member is set to zero. (12) ensures that the speed $y_{t,j}$ of the slower person is set to $y_{t,j} = v_{t,j}$. (13) makes use of the help variable $y_{t,j}$ and ensures that the two team members walk the full distance d. The final equations (17) and (18) define the values of the different variables.

The effort for developing the presented MILP model was quite high. At the beginning, we tried using discrete integer decisions variables that describe the time points when the two members have a beer (in contrast to the presented decision variables which are binary and time-dependent). However, using discrete integer decision variables did not allow us to model the other constraints appropriately. Furthermore, we had to introduce additional help variables $y_{t,j}$ to linearize the model and to avoid non-linearities in the problem formulation. Overall, the effort for developing the MILP model was high and it took several days to formulate and verify the complete model.

4. GENETIC ALGORITHM

Genetic algorithms (GA) [5] are nature-inspired search techniques used for optimization problems where each candidate solution or individual is represented by a genome. The important design variables for GAs are the used fitness function and the encoding which determines the possible mutation and crossover operators. To choose a proper representation and corresponding search operators is important for high-quality GAs [8].

In GAs, a fitness function evaluates the individuals according to the objective. The population of candidate solutions iteratively goes through the process of selection, recombination, and mutation. In the selection step, the search focuses on promising areas of the search space. Selection usually discards individuals of low quality and keeps solutions with high fitness values in the population. Recombination chooses promising solutions and recombines them to form new solutions. The last step in each generation is the mutation of individuals by changing parts of their genomes. These steps are iteratively repeated until a pre-defined termination criteria (e.g. number of iterations or a minimum fitness) is reached.

For the problem at hand, we choose the following GA design. We use a simple standard GA [5] with population size N, fitness-proportional selection, standard one-point crossover, and bit-flipping mutation. We encode each solution using a genome that consists of a vector of n^b tuples. Each of the $i \in \{0, \dots, n^b\}$ tuples consist of an integer number $t \in \{0, \dots, t_{max}\}$ which indicates the time t a team member is having a drink and a binary variable $j \in \{0, 1\}$ which indicates which of the two team members is having the drink at that time. The one-point crossover operator is applied to two randomly chosen solutions, randomly selects a cutting point, and exchanges the sub-strings between both parental solutions. Mutation randomly changes t_i or j_i with some probability p_{mut}. The initial population of the GA is generated randomly assigning random values to the t_i and j_i. As we assumed in Section 2.1 that only one person can consume a beer at time t invalid solutions can occur if $t_i = t_l$, where $i \neq l$. Therefore, we use an additional repair operator that ensures that only valid solutions can be created in the initial population and as result of the crossover and mutation operator.

The size of the genotypic search space is $\binom{2t_{max}res}{n^b}$ if we assume that the team consists of two persons. res (in $1/min$) is the resolution that is used for discretizing the time t. The size of the GA search space is lower than the size of the search space for the MILP model from Section 3, which is $2^{2t_{max}res}$, but also contains infeasible solutions. Using the additional constraints defined in Section 3 and considering only feasible solutions reduces the size of the MILP search space to $\binom{2t_{max}res}{n^b}$, which is the same as for the GA.

For the GA, the calculation of the fitness is done according to the models described in Sections 2.2 and 3. Based on the models we implemented a discrete event simulator calculating the BAL $p_{t,j}$ and weight c_t of the case at every time t (the standard resolution was minutes). Based on these values the speeds $v_{t,j}$ are determined at each time point. As before, the BAL $p_{t,j}$ of person j depends on the previous BAL $p_{t-1,j}$ and Δp_j (compare (7)) and the number of drinks in the previous hour. The weight of the case is calculated as in (5) using the initial weight of the case and subtracting the weight of one beer for every drink consumed by one of the two team members. The speed is calculated according to (8) and depends on the speed decrease caused by the BAL and the remaining weight of the case.

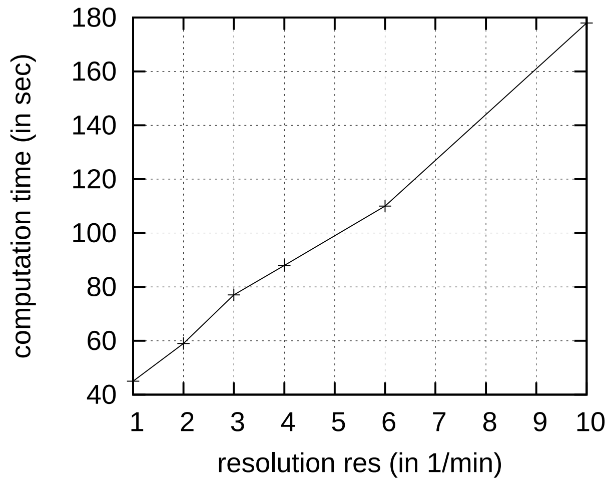

Figure 1: We show for 1,000,000 simulations the overall mean computational effort (in sec) over the used resolution res (in 1/min).

The resolution of the simulation is usually set to $res = 1$ (resolution of one minute). The resolution can be increased with linear complexity as to use a resolution of $res = 60$ (shortest time interval is one second) 60 times the simulation steps are necessary that are used for $res = 1$. Figure 1 shows the computation time (in seconds) necessary for performing 1,000,000 simulations of different strategies (solutions) using different resolutions res (varying between six seconds and one minute). For the simulations, we used the same computer as described in Section 5.

Although we assume in all the following experiments (and in the MILP model from Section 3) that always the case is carried by both people (8), the used discrete event simulator and the GA can easily be modified or extended such that only one person of the team can carry the case at time t. We have performed some experiments using the GA to determine optimal strategies allowing that the case is carried either by both persons or by one person alone. The results obtained by the GA show that such "weight-shifting strategies" do not significantly reduce the time to reach the goal. We do not present results for this extended version of the game as we could not extend the MILP from Section 3 such that the additional possibilities (either one or two persons carry the case) are modeled appropriately and thus we would have no benchmark for the GA and would not be able to verify that the GA finds the optimal solution for the problem.

The effort for developing a GA for the problem at hand was pretty low. We experimented with different intuitive representations like the one presented here and a different possibility which uses binary decision variable as for the MILP model. We have chosen the encoding presented here as using integer decision variables (which indicates when the team members are having a drink) was intuitive and easy to implement. The discrete event simulator as well as the used GA are quite simple and straightforward and both have been implemented in a few hours using Java. Overall the effort for designing and implementing the GA (including the discrete event simulator) was a few days.

5. COMPUTATIONAL EXPERIMENTS AND RESULTS

We evaluate the performance of the GA and a MILP solver for solving the problem defined in Section 2. The maximal time allowed to reach the goal is $t_{max} = 180min$ ($t \in \{0, \dots, 180\}$). For the experiments we use either a resolution of $res = 1$ (resolution of one minute) or $res = 60$ (resolution of one second). Therefore, we have $2 \times 180 \times res$ decision variables $x_{t,j}$ for the MILP model from Section 3 and the cardinality of the t_i for the GA is $t_{max} \times res$. We assume that males have a body weight of 80kg and females a body weight of 55kg. Furthermore, we assume a maximum running speed (with the empty case) of $v_j^{max} = 6km/h$. The speed decrease per kg additional weight is set either to $\Delta v_j^w = 1/4 \frac{km}{hkg}$ or to $\Delta v_j^w = 1/6 \frac{km}{hkg}$ and the speed decrease per BAL (in ppt) is set to $\Delta v_j^p = 1 \frac{km}{hppt}$. Each beer has a volume share of 5% alcohol.

All calculations were performed under Linux on a Xeon 3Ghz with 2 GB of RAM. The MILP was solved by using CPLEX 9.0, a solver for MILPs from ILOG. The input files for CPLEX were generated by a self-written Java program. The performance of CPLEX for the considered test problem was disappointing. When determining the optimal strategy for the test scenario with a resolution of $res = 1$, the initial feasible solution (goal reached after 180min) could not be improved after a calculation time of 24 hours when we aborted the optimization. Therefore, the MILP model from Section 3 could not be solved by CPLEX in reasonable time.

When increasing the accuracy from $res = 1$ to $res = 60$, the number of variables and the number of constraints increases by a factor of 60. For $res = 60$, the input file for CPLEX containing the constraints is larger than 2 GB so we could not read the file into CPLEX as the size of the file exceeds the available memory of the computer. As CPLEX was not able to solve the problem for $res = 1$ we assume that we also get no results using a higher resolution.

The GA is implemented in Java and uses the configuration described above. In all runs we use an population size of $N = 1,000$, a crossover probability of $p_{cross} = 1$, and a (pretty high) mutation probability $p_{mut} = 0.1$. A GA run is stopped after $t_{conv} = 1,000$ generations. For $res = 1$ the mean running time of the GA (including the time necessary for the discrete event simulator) is about 45 seconds on our test server. We performed 20 runs for the GA and the average fitness of the best found solution is 151.5. This means when using the optimal strategy for the game the team arrives at the goal after about 150 minutes.

Although we have not been able to solve the problem using CPLEX, we can use the MILP model to verify that the solution that has been found by the GA is optimal. In the optimal solution found by the GA the team reaches the goal after 151 minutes. Therefore, $t_i = 0$ for $i > 152$. When

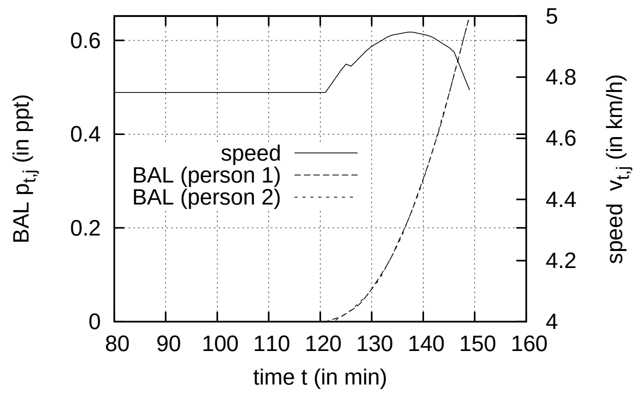

Figure 2: We show the BAL $p_{t,j}$ and the speed $v_{t,j}$ over the time t for $\Delta v^w = 1/6$ and two males. The resolution was set to $res = 1$.

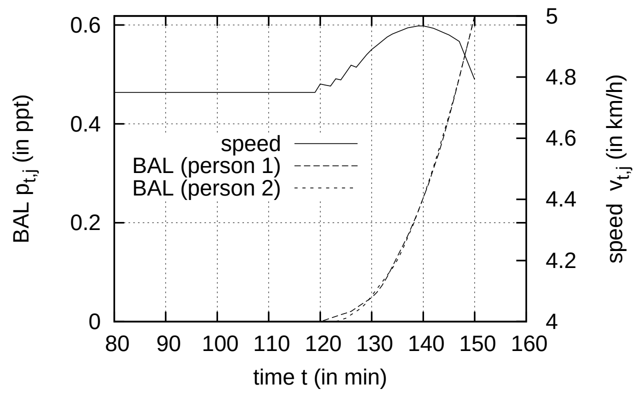

Figure 3: We show the BAL $p_{t,j}$ and the speed $v_{t,j}$ over the time t for $\Delta v^w = 1/6$ and two males. The resolution was set to $res = 60$.

adding this additional constraint to the MILP ($r_{152} = 0$), CPLEX can solve the problem in a few minutes and confirms that the solution found by the GA is the optimal solution for the problem.

The following figures discuss the properties of the optimal strategy found by the GA. Figure 2 shows the BAL $p_{t,j}$ and the speed of the team $v_{t,j}$ over the time t. We assume two males and $\Delta v_j^w = 1/6 \frac{km}{hkg}$. We only show results for $t > 80min$ as no drinks are consumed before and thus the speed and the BAL remains constant. The plots reveal that the optimal strategy for the game is to consume the drinks at the end of the run with increasing frequency. As a result of consuming one drink the speed $v_{t,j}$ slightly increases (due to the lower weight c_t of the case). However, with some delay the speed is reduced again due to the increasing BAL. The results indicate that the speed when reaching the goal is about the same speed before starting having some drinks. Furthermore, both team members have the same number of drinks so their BALs $p_{t,j}$ are about the same. Figure 3 shows the optimal strategy found by the GA for the same scenario but uses a higher resolution of $res = 60$. The results show that there are only minor differences between $res = 1$ (Figure 2) and $res = 60$ (Figure 3). Therefore, using $res = 1$ is sufficient for finding optimal strategies.

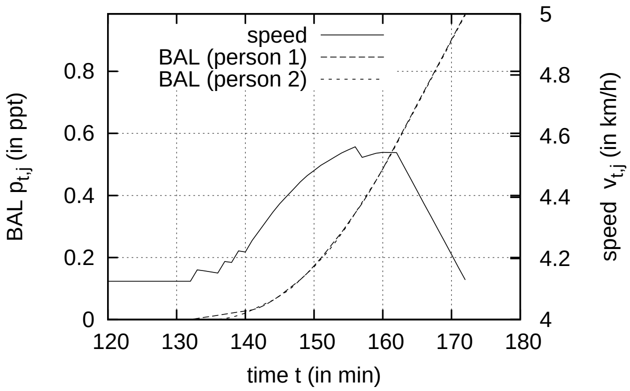

Figure 4: We show the BAL $p_{t,j}$ and the speed $v_{t,j}$ over the time t for $\Delta v^w = 1/4$ and two males. The resolution was set to $res = 1$.

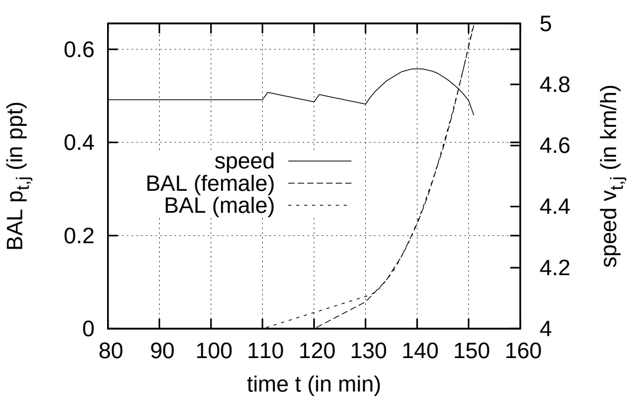

Figure 5: We show the BAL $p_{t,j}$ and the speed $v_{t,j}$ over the time t for $\Delta v^w = 1/4$ and a team consisting of one female with a body weight of 55kg and one male with a body weight of 80kg.

Figure 4 shows results for the same setting using a different $\Delta v_j^w = 1/4$. Therefore, the weight of the drinks stronger reduces the maximum speed v_j^{max} of the two team members and the time necessary to reach the goal increases to about $173min$. The plots reveal that, as expected, the speed of the team is lower than for $\Delta v_j^w = 1/6$ as the weight slows down the team to a larger degree. Furthermore, the two persons have to start having the drinks earlier. Again, the speed of the team when reaching the goal is about the same as before having some drinks. Furthermore, both persons have about the same number of drinks.

Figure 5 shows the optimal strategy for a team of two different persons. We use $\Delta v_j^w = 1/6$ and assume a team that consists of a male with a body weight of $80kg$ and a female with a body weight of $55kg$. The optimal strategy allows the team to reach the goal after 153min. The plots reveal that the optimal strategy is to have the same BAL for both persons. Therefore, the male has to have a larger number of drinks in comparison to the female and both have to start consuming drinks earlier.

6. CONCLUSIONS

This paper applied two different optimization approaches, a MILP solver and a simple GA, to a problem denoted as "beer-run". The problem was chosen as it has not been yet studied before and no information about the properties of the problem nor its difficulty exists. The paper related the problem to similar problems from other domains and developed a MILP model of the problem as well as a GA. The development of the different solution approaches (MILP versus GA) showed that the effort for developing the MILP was higher than the effort for the GA. Studying the performance of the two different approaches revealed that simple variants of the problem that can be solved to optimality by the GA in a few seconds can not be solved by the MILP solver with reasonable effort.

Several issues can be learned from the paper. First, developing the two optimization methods for the problem and examining their performance shows that the effort for developing the MILP model was higher than designing a simple GA. The experiences made during the development of the different approaches confirmed the ease of use of metaheuristics in comparison to classical linear programming approaches. Second, the performance of the two approaches are in contrast to the effort for developing them as a state-of-the-art MILP solver can not solve the MILP problem. In contrast, the problem can easily be solved by a simple GA in a few seconds. Furthermore, for the GA approach extensions and modifications of the objective function or the problem can easily be considered whereas it is difficult to incorporate them in the MILP model. Third, the paper developed a MILP model and a GA approach for the problem at hand. Although the problem is some kind of artificial it is related to other logistics problem. Furthermore, the proposed models can be of use for solving problems where some kind of goods like poison or polluted air must be consumed and the consumption of such goods has negative effects (e.g. the inhalation of polluted air or poison reduces the speed of a person) and positive effects (e.g. when moving in polluted areas inhaling some amount of polluted air is necessary to survive). Finally, optimal strategies for the "beer-run" have been developed. The strategies show that the optimal strategy is to have the drinks at the end of the run and such that all team members have the same blood alcohol level during the run.

Besides other things, the proposed study roughly compares the effort that is necessary for developing optimization systems for problems of unknown properties and complexity. In this paper the observation that the development of efficient metaheuristics is faster than the development of exact methods for new problems is only based on rough approximations for the effort that has to spent for developing the different approaches. No quantitative models for measuring the development costs have been used. It would be interesting to examine expenditure models (like xyz) from the software engineering domain and to adapt such models to the development of optimization systems. Using quantitative models would allow us to guess a priori the expected effort for solving a new problem using a particular optimization technique.

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